

What is Integration by Parts?

Integration by parts is a technique based on the product rule for differentiation that allows us to exchange one integral for another.

Formula:

The integration-by-parts formula for the integral involving two functions is:

$$\int u \, dv = u * v - \int v \, du$$

The most difficult part of using the integration by parts technique is deciding which function will be your u and which function will be your v . Sometimes, it'll be a matter of trial and error, but there is a way to mitigate this by using the acronym **LIATE**.

The acronym **LIATE** stands for **Logarithmic Functions, Inverse Trigonometric Functions, Algebraic Functions, Trigonometric Functions, and Exponential Functions.**

ORDER	MEANING
L	Logarithmic
I	Inverse Trigonometric
A	Algebraic
T	Trigonometric
E	Exponential

The first function that appears will be your first choice for u . For example, if an integral contains a logarithmic function and an algebraic function, the u will be the logarithmic function because the L comes before the A in the table. The remaining function is going to be the dv .

Steps for Integration by Parts:

- Choose your u and dv .
 - Once you have the u , you will need to derive it to get du
 - Once you have the dv , you will need to integrate it to get the v
- Apply the formula.

Example 1:

$$\int x e^{2x} dx$$

Example 1 Solution:

$$1. \quad u = x \quad \rightarrow \quad du = 1 dx$$

$$dv = e^{2x} dx \quad \rightarrow \quad v = \int e^{2x} dx = \frac{1}{2} e^{2x}$$

$$\begin{aligned}
 2. \int x e^{2x} dx &\rightarrow u * v - \int v du = (x) \left(\frac{1}{2} e^{2x} \right) - \int \frac{1}{2} e^{2x} dx \\
 &= \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx \\
 &= \frac{1}{2} x e^{2x} - \frac{1}{2} \left(\frac{1}{2} e^{2x} \right) \\
 &= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C
 \end{aligned}$$

Example 2:

$$\int x \sin(x) dx$$

Example 2 Solution:

$$1. u = x \rightarrow du = 1 dx$$

$$dv = \sin(x) dx \rightarrow v = \int \sin(x) dx = -\cos(x)$$

$$\begin{aligned}
 2. \int x \sin(x) dx &\rightarrow u * v - \int v du = (x)(-\cos x) - \int (-\cos x) dx \\
 &= -x \cos(x) + \int \cos(x) dx \\
 &= -x \cos(x) + \sin(x) + C
 \end{aligned}$$

References:

Strang, Gilbert, and Edwin Herman. "3.1 Integration by Parts." *OpenStax*, 30 Mar. 2016, openstax.org/books/calculus-volume-2/pages/3-1-integration-by-parts. Accessed 19 May 2023.

Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.