Finding Asymptotes

An **asymptote** is a line that a curve approaches but never touches. A line where the graph of a function converges is known as an asymptote.

Three Common Types of Asymptotes:

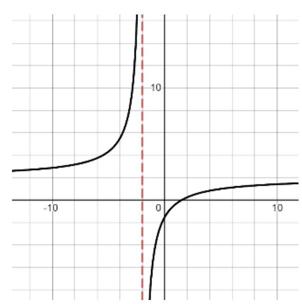
1. **Vertical** – A vertical line that appears to correspond with the graph of a function but never really touches the curve. It is in the form of $x = k$, where k is the real number to which the function approaches to.

Example: Find the vertical asymptote of $f(x) = \frac{4x-6}{2x+4}$

1. Set the denominator of the function equal to zero:

$$2x + 4 = 0$$

2. Solve for x:



$$2x + 4 = 0$$

$$-4 = -4$$

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$x = -2$$

VA = −2

2. **Horizontal** – A horizontal line that appears to correspond with the graph of a function but never really touches the curve. It is in the form of $y = k$, where k is the real number to which the function approaches to.

NOTE: There are three rules to solve for a horizontal asymptote.

The horizontal asymptote depends on the degree of the numerator and the degree of the denominator.

$$f(x) = \frac{ax^n + \cdots}{ax^m + \cdots}$$

If $n < m$, the horizontal asymptote is the x-axis. $(y = 0)$	If $n = m$, the horizontal asymptote is $y = \frac{a}{b}$.	If $n > m$, there is no horizontal asymptote. (There is an oblique asymptote)
Example: $f(x) = \frac{4x+2}{x^2+2}$ $\frac{4x^{1} + 2}{x^{2} + 2}$ $y = 0$ The degree on the denominator is bigger than the one on the numerator; therefore, the HA is $y = 0$	Example: $f(x) = \frac{4x^2+2}{x^2+2}$ $\frac{4x^{2} + 2}{x^{2} + 2}$ $y = 4/1 = 4$ The degree on the denominator equals the degree of the numerator; therefore, the HA is $y = 4$ the leading coefficient ratio.	Example: $f(x) = \frac{4x^2+2}{x+2}$ $\frac{4x^{2} + 2}{x^{1} + 2}$ The degree on the denominator is less than the one on the numerator; therefore, there is no HA.

3. **Slant/Oblique** – A slanted line that appears to correspond with the graph of a function but never really touches the curve.

NOTE: A slant asymptote is typically resulted when there is no horizontal asymptote. To know if there is a slant asymptote, the degree of the numerator must exceed the degree of the denominator by exactly one.

Example: Find the slant/oblique asymptote of $f(x) = \frac{x^2+5x+6}{x+3}$
1. Using long division, divide the numerator by the denominator.

$$\begin{array}{r} \boxed{x+2} \\ x+3\overline{)\,x^2+5x+6} \\ -(x^2+3x) \\ \hline 0+2x+6 \\ -(2x+6) \\ \hline 0 \end{array}$$

The equation $y = x + 2$ is the slant asymptote.

References:

Asymptotes - horizontal, vertical, slant (oblique). Cuemath. (n.d.). https://www.cuemath.com/calculus/asymptotes/

Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.