Ellipse

An **ellipse** is formed when a plane cuts through a right circular cone. An ellipse is the set of all points *(x,y)* in a plane such that the sum of their distances from two fixed points is a constant. Each fixed point is called a **focus** (plural: *foci*).

Standard form of an ellipse

With a center (0,0) and the major axis located on the x-axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

An ellipse has a

- **Center**
- **Vertices**
- **Co-vertices**
- **Foci**

- A **center** of an ellipse without shifts in the $x^2$and y^2 will have a center of (0,0).
- A **major axis** will be located on the **x-axis** when ***a* > *b***, ***a*** being in the **denominator of x^2.** If the **major axis** is located on the **x-axis, then** the **vertices** will be located on the **x-axis** and the **co-vertices** will be located on the **y-axis**.
- **Vertices** of a standard form ellipse are ($\pm a$, 0), ***a*** is the **square root of a^2**.
- **Co-vertices** of a standard form ellipse are $(0, \pm b)$, ***b*** is the **square root of b^2**.
- The coordinates of a **foci** will be $(\pm c, 0)$, where $c^2 = a^2 - b^2$, c is equal to the square root of c. **Foci** will **always be** on the **major axis**.

Ellipse

Example: Standard form ellipse

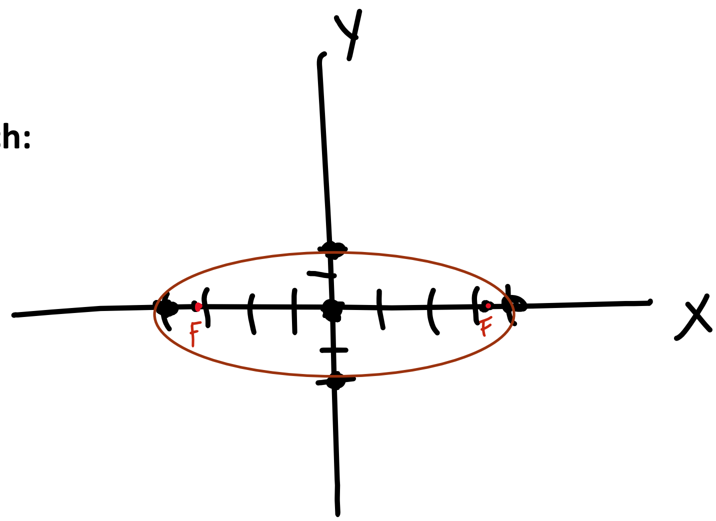
$\frac{x^2}{16} + \frac{y^2}{4}$ =1

Since there are no shifts in the x^2 and y^2, which would look like $(x-h)^2$ and/or $(y-k)^2$, h and k being constants, **our center is (0,0)**.

Our **major is located on the x-axis** since square root of the number under the **denominator of x^2** is **greater** than the square root of the number under the **denominator of y^2.**

Center: (0,0)

Sketch:



Vertices: (-4,0), (4,0)

$(\sqrt{a^2}, 0)$ → $(\pm 4,0)$

Co-vertices: (0,-2), (0,2)

(0, $\sqrt{b^2}$) → (0, ± 2)

Foci: (-2$\sqrt{3}, 0$), (2$\sqrt{3}, 0$)

$(\pm c, 0)$ → $c^2 = a^2 - b^2$ → 16-4= 12 → $c = \pm\sqrt{12} \rightarrow \pm\sqrt{4*3} \rightarrow \pm 2\sqrt{3}$

$(\pm 2\sqrt{3}, 0)$

$\pm 2\sqrt{3}$ is approximately $\pm$3.464

Example: Ellipse with shifts in x^2 and y^2

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}$=1

$$\frac{(x-3)^2}{16}+\frac{(y+2)^2}{4}$$

This is the same example from the top but now with **shifts** in the **x** and **y** which **change our center to (h, k)**. Our **major axis** will continue being on the **x-axis** since the **denominator of x^2** is still **greater** than the **denominator of y^2**.

When finding our **vertices, co-vertices, and foci** we will have to **keep our shifts in mind.**

Center: (h, k) → (3, -2)

Vertices: (-1, -2), (7, -2)

($h \pm a, k$),where ***h*** is equal to 3 and ***a*** is equal to $\sqrt{a^2}$= ± 4,

($3 \pm 4, -2$)

Co-vertices: (3, -4), (3, 0)

(h, k$\pm b$), where ***k*** is equal to -2 and ***b*** is equal to $\sqrt{b^2}$=± 2

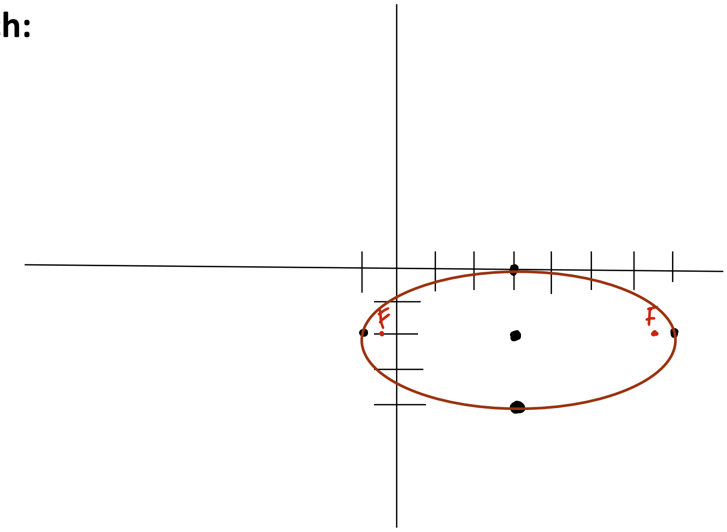
(3, -2$\pm$2)

Foci: (3-2$\sqrt{3}$, -2), (3+2$\sqrt{3}$, -2) ≈ (-.464, -2), (6.464, -2)

$(h \pm c, \mathrm{k})$, where ***h*** is equal to 3 and ***c*** is equal to the square root of c^2 and $\boldsymbol{c^2} = \boldsymbol{a^2} - \boldsymbol{b^2}$

$(3 \pm 2\sqrt{3}$, -2)

Sketch:



Example: Vertical ellipse (change in major axis)

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

- Remember that to identify our major axis we need to identify where *a>b*. In this example, the **major axis** is located on the **y-axis**. This is because the **denominator** of **y²** is **greater** than the **denominator** of **x²**. Since the **foci** is always located on the **major axis** as well, the **foci** will also be located on the **y-axis**. The co-vertices will be located on the x-axis.

Center: (0,0)

Vertices: $(0, \pm 4)$

Co-vertices: $(\pm 2, 0)$

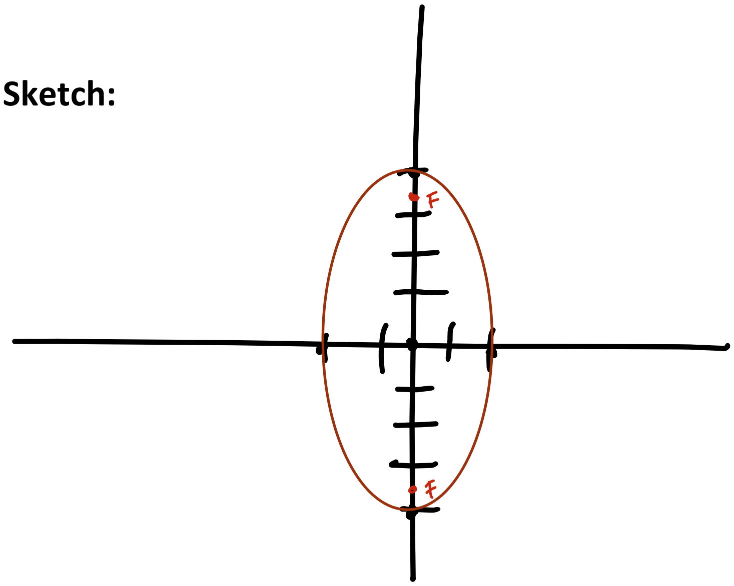
Foci: $(0, \pm 2\sqrt{3})$

$c^2 = a^2 - b^2$

$c^2 = 16 - 4 = 12$

$c = 2\sqrt{3}$

Sketch:



References:

Ambramson, J. (2023). *OpenStax*. college algebra open stax. Retrieved March 29, 2023, from https://assets.openstax.org/oscms-prodcms/media/documents/CollegeAlgebra-OP.pdf

College Algebra OpenStax Section 8.1 Ellipse

Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.